Exercise 30

Explain, in terms of linear approximations or differentials, why the approximation is reasonable.

$$\sqrt{4.02} \approx 2.005$$

Solution

Compute the derivative of $y = \sqrt{x}$.

$$\frac{dy}{dx} = \frac{d}{dx}(\sqrt{x})$$

$$= \frac{d}{dx}(x^{1/2})$$

$$= \frac{1}{2}x^{-1/2}$$

Consequently, the differential of $y = \sqrt{x}$ is

$$dy = \frac{1}{2}x^{-1/2}\,dx.$$

In order to estimate $\sqrt{4.02}$, set $x = 4$ and $dx = 0.02$.

$$dy = \frac{1}{2}(4)^{-1/2}(0.02) = \frac{1}{200} = 0.005$$

Note that dy here is the vertical distance from the function's actual value at $x = 4$ to the linear approximation's value at $x = 4.02$.

$$\sqrt{4.02} \approx \sqrt{4} + 0.005 = 2.005$$